# Bound, virtual and resonance S-matrix poles from the Schrödinger equation 

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A general method, which we call the potential S-matrix pole method, is developed for obtaining the S-matrix pole parameters for bound, virtual and resonant states based on numerical solutions of the Schrödinger equation. This method is well-known for bound states. In this work we generalize it for resonant and virtual states, although the corresponding solutions increase exponentially when $r \rightarrow \infty$. Concrete calculations are performed for the $1^{+}$ground state of ${ }^{14} \mathrm{~N}$, the resonance ${ }^{15} \mathrm{~F}$ states $\left(1 / 2^{+}, 5 / 2^{+}\right)$, low-lying states of ${ }^{11} \mathrm{Be}$ and ${ }^{11} \mathrm{~N}$, and the subthreshold resonance in the proton-proton system. We also demonstrate that in the case of the broad resonances their energy and width can be found from the fitting of the experimental phase shifts using the analytical expression for the S-matrix. We compare the Smatrix pole and the R-matrix for broad resonances in the ${ }^{14} \mathrm{O}-\mathrm{p}$ and in ${ }^{26} \mathrm{Mg}-n$.

As example, in Table I we present the results of the calculations of the resonance states $1 / 2^{ \pm}$and $5 / 2^{+}$in ${ }^{11} \mathrm{~N}$. In the 6 -th and 7 -th columns are shown the calculated energies and single-particle proton partial resonance widths.

TABLE I. Energies and widths calculated for low-lying levels of ${ }^{11} \mathrm{~N}$ by S-matrix pole method. The Coulomb radius $r_{C}=1.1 \mathrm{fm}\left(r_{l s}=r_{0}, a_{\mathrm{ls}}=a\right)$.

| J | $r_{0}$ <br> $(\mathrm{fm})$ | $a$ <br> $(\mathrm{fm})$ | $V_{0}$ <br> $(\mathrm{MeV})$ | $V_{l s}$ <br> $(\mathrm{MeV})$ | $E_{\text {sp }}$ <br> $(\mathrm{MeV})$ | $\Gamma_{\text {sp }}$ <br> $(\mathrm{MeV})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 2^{+}$ | 1.20 | 0.753 | 57.057 | 0 | 1.014 | 0.843 |
|  | 1.22 | 0.713 | 57.057 | 0 | 1.039 | 0.881 |
|  | 1.25 | 0.650 | 57.057 | 0 | 1.081 | 0.944 |
|  | 1.27 | 0.607 | 57.057 | 0 | 1.112 | 0.993 |
|  | 1.29 | 0.562 | 57.057 | 0 | 1.146 | 1.048 |
| $1 / 2^{-}$ | 1.20 | 0.819 | 37.505 | 6.0 | 1.919 | 0.944 |
|  | 1.22 | 0.760 | 37.505 | 6.0 | 1.991 | 0.963 |
|  | 1.25 | 0.650 | 37.505 | 6.0 | 2.134 | 0.996 |
|  | 1.27 | 0.545 | 37.505 | 6.0 | 2.284 | 1.024 |
|  | 1.28 | 0.451 | 37.505 | 6.0 | 2.426 | 1.047 |
|  | 1.20 | 0.753 | 57.057 | 7.13 | 3.672 | 0.959 |
|  | 1.22 | 0.713 | 57.057 | 6.222 | 3.719 | 0.927 |
|  | 1.25 | 0.650 | 57.057 | 4.743 | 3.793 | 0.878 |
|  | 1.27 | 0.607 | 57.057 | 3.671 | 3.845 | 0.847 |
|  | 1.29 | 0.562 | 57.057 | 2.520 | 3.900 | 0.8167 |

